



# Optimization-based Estimation of Environment Obstacles from Human Demonstration

## Using Control Lyapunov Function and Control Barrier Functions

Shafagh Keyvanian, HoJin Choi, Minku Kim

### Introduction

#### Motivation:

- Robots often rely on sensors for obstacle detection.
- Leveraging expert demonstrations without sensors can enhance robot obstacle recognition in cases of obstacle occlusion.
- Enabling recognition of non-physical obstacles such as unsafe or non-favorable regions in the environment.

#### Contributions:

- Modeling human behavior around obstacles: Learning  $\alpha$  and  $\gamma$  using probabilistic model of expert demonstrations
- Learning position and radius of obstacles:  $x_{obs}$  and  $r_{obs}$  using *all* human demonstrations (including non-expert demos)
- Multiple obstacles estimation

#### Background:

- *Control Lyapunov Function* for stability assurance and *Control Barrier Functions* for safety assurance, as constraints of a QP optimization

$$u(x) = \arg \min_{(u,\delta) \in R^{m+1}} \frac{1}{2} u^T H(x) u + p \delta^2 \quad (\text{CLF} - \text{CBF} - \text{QP})$$

$$\text{s.t. } L_f V(x) + L_g V(x) u \leq -\gamma(V(x)) + \delta$$

$$L_f h(x) + L_g h(x) u \geq -\alpha(h(x))$$

### Data Construction

#### 1. 2D optimization-based simulation

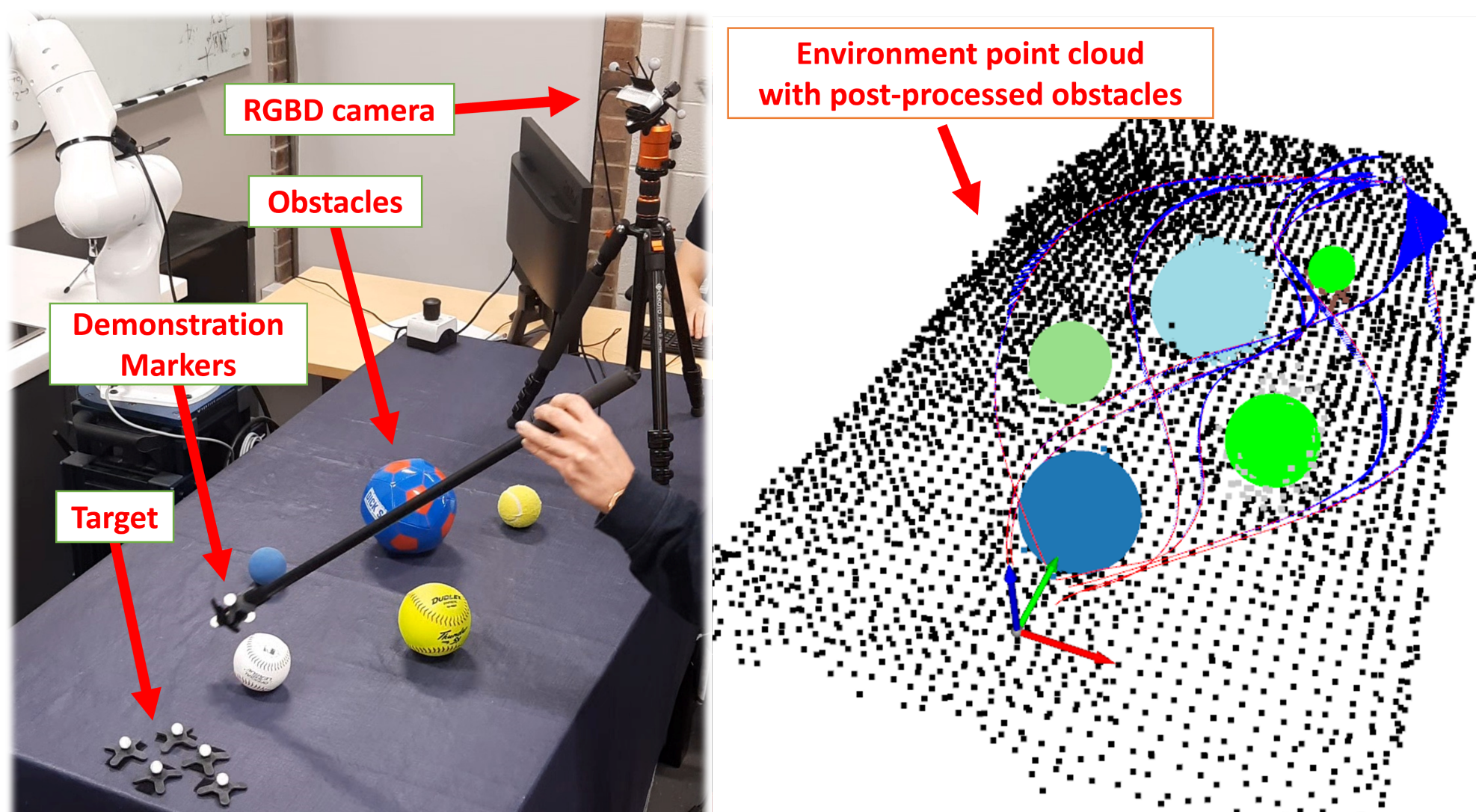
Trajectory with input constraints and cost for velocity and acceleration using non-linear optimization from Drake

#### 2. 2D human-like demonstrations with mouse

#### 3. 2D and 3D real human demonstrations

Motion Capture system to record demonstrations

RGBD camera to create point clouds for environments and obstacles



### Methods

#### ► Problem Assumptions

1. Obstacle shapes can be decomposed into multiple spheres.
2. Expert trajectories lead to a stable target without collision.

#### ► Problem Formulation

$$u_k = x_{k+1} - x_k$$

$$\text{Lyapunov function: } V(x) = (x_{\text{target}} - x)^2$$

$$\text{Barrier function: } h(x) = (x - x_{\text{obstacle}})^2 - r_{\text{obstacle}}^2$$

$$\alpha(h(x)) = \alpha h(x) \text{ and } \gamma(V(x)) = \gamma V(x)$$

#### CLF-CBF-QP:

$$u_{k,\text{estimated}} = \arg \min_{(u,\delta) \in R^{m+1}} \|x_{k+1} - x_k\|_2^2 + p \delta^2$$

$$\text{s.t. } 2(x_{\text{target}} - x)u \leq -\gamma(x_{\text{target}} - x)^2 + \delta$$

$$2(x - x_{\text{obstacle}})u \geq -\alpha((x - x_{\text{obstacle}})^2 - r_{\text{obstacle}}^2)$$

#### Loss function:

$$\text{loss}(u_{\text{estimated}}, u_{\text{demo}}) = 1 - \cos(u_{\text{estimated}}, u_{\text{demo}})$$

#### ► Step 1: Learning $\alpha$ and $\gamma$

##### Modeling human-like obstacle avoidance behavior

- *GMM-GMR\** used to learn human-like trajectory around obstacles from a few expert demonstrations
- The resulting regression function used for  $x_k$  and  $u_k$
- Obstacles known,  $\alpha$  and  $\gamma$  unknown
- Explore  $\alpha$  and  $\gamma$  until Loss value converges.

#### ► Step 2: Obstacle Estimation

- All demonstrations data used for  $x_k$  and  $u_k$
- $\alpha$ ,  $\gamma$  learnt in step 1, obstacles' position and size unknown
- Explore obstacles until Loss value converges

\* **GMM-GMR:** Trajectory modeling from expert demonstrations  
**GMM:** Mixture of Gaussian Functions

$$P(x, y) = \sum_{k=1}^K \pi_k \mathcal{N}(x, y, \mu_k, \Sigma_k)$$

$$\mu_k = \begin{bmatrix} \mu_{x,k} \\ \mu_{y,k} \end{bmatrix}, \Sigma_k = \begin{bmatrix} \sum x, k & \sum xy, k \\ \sum yx, k & \sum y, k \end{bmatrix}, \sum_{k=1}^K \pi_k = 1$$

**GMR:** Regression function

$$y(x) = \sum_{k=1}^K \beta_k (\mu_{k,y} + \Sigma_{k,yx} (\Sigma_{k,x})^{-1} (x - \mu_{k,x}))$$

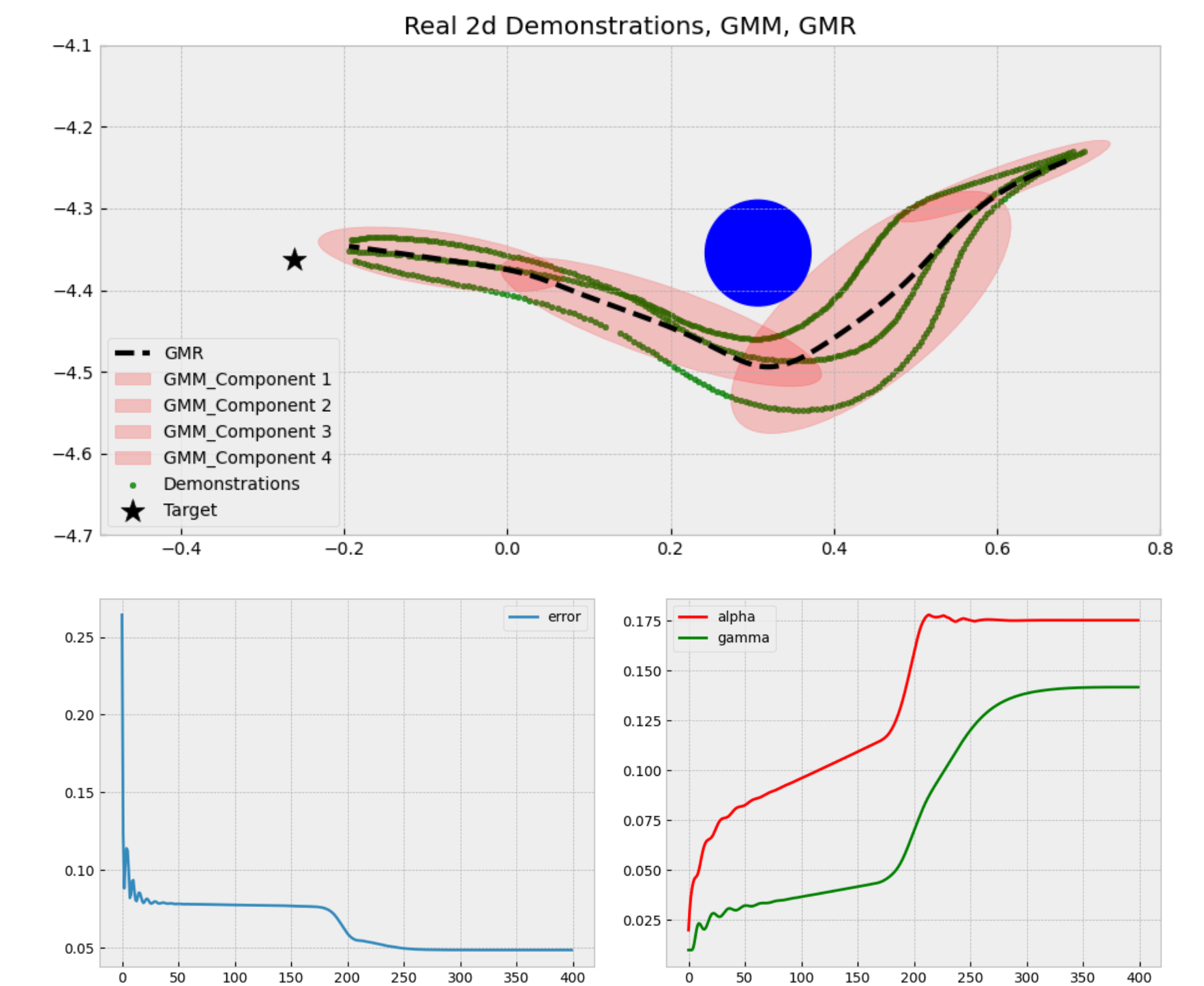
$$\beta_k = \beta_k \frac{\pi_k \mathcal{N}(x, \mu_{k,x}, \Sigma_{k,x})}{\sum_{i=1}^K \pi_i \mathcal{N}(x, \mu_{k,i}, \Sigma_{k,i})}$$

$K, \pi_k, \mu_k, \Sigma_k$ : # Gaussian kernels, prior probability, mean, covariance matrices

### Results

#### ► Step 1: Learning $\alpha$ and $\gamma$

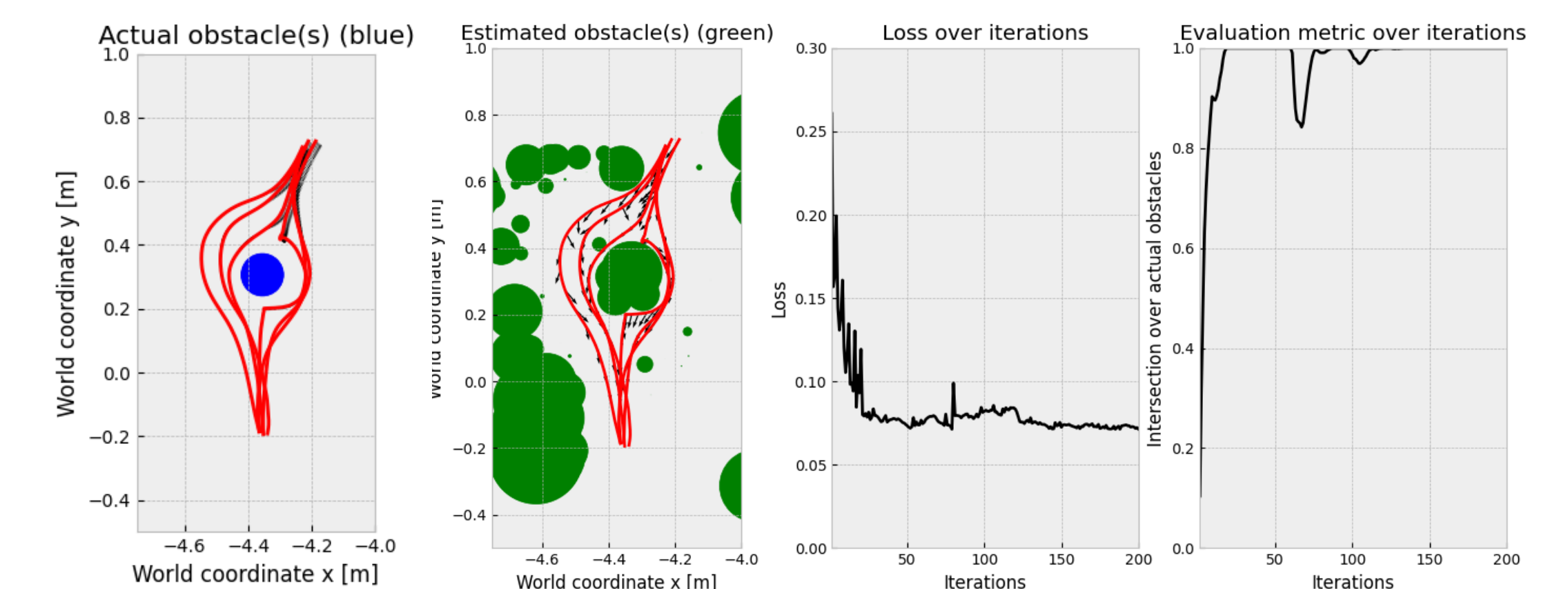
- *GMR* Regression function from a few expert demonstrations
- Explore  $\alpha$  and  $\gamma$  until convergence.



#### ► Step 2: Obstacle Estimation

- All demonstrations data used as  $x_k$  and  $u_k$
- Explore  $x_{\text{obstacles}}$  and  $r_{\text{obstacles}}$  until convergence.
- Evaluation Metric: Intersection over actual obstacles  $\frac{A \cap B}{A}$  ( $A$ : Actual obstacles' area,  $B$ : Predicted obstacles' area)

Results for environment with one obstacle:



Results for environment with multiple obstacles:

